PHYSICS ACADEMY
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MATHEMATICAL PHYSICS

ASSIGMENT- VECTOR ANALYSIS

"CSIR-NET/JRF DEC-2021"

For –

- ☞ **CSIR-NET/JRF**
- ☞ **GATE / JEST**
- ☞ **TIFR / BARC**
- ☞ **All Ph.D. Entrance Exams**

"We Believe In Quality Education"

LEVEL-1

1. $\nabla^2\left(\frac{1}{r}\right)$ is:

(a) 0 (b) $-\delta(r)$ (c) $-4\pi\delta(r)$ (d) $4\pi\delta(r)$

2. $\iiint \nabla^2\left(\frac{1}{r}\right) dV \quad r \neq 0$ is:

(a) 0 (b) -4π (c) 4π (d) 1

3. A vector field id defined everywhere as $\vec{F} = \frac{y^2}{L}\hat{\imath} + z\hat{k}$. The net flux of $\vec{F}$ associated with a cube of side I, with one vertex at the origin and sides along the positive X, Y and Z axes is:

(a) L^3 (b) $4L^3$ (c) $8L^3$ (d) $10L^3$

4. If a vector field $\vec{F} = x\hat{\imath} + 2y\hat{\jmath} + 3z\hat{k}$, then $\vec{\nabla} \times (\vec{\nabla} \times \vec{F})$ is :

(a) 0 (b) $\hat{\imath}$ (c) $2\hat{\imath}$ (d) $3\hat{k}$

<u>Common Data for Q.5 & Q.6-</u>

Consider the vector $\vec{V} = \frac{\vec{r}}{r^3}$

5. The surface integral of this vector over the surface of a cube of size and centered at the origin.

(a) 0 (b) 2π (c) $2\pi a^3$ (d) 4π

6. Which of the following is not correct?

(a) Value of the line integral of this vector around any closed curve is zero.
(b) This vector can be written as the gradient of some scalar function.
(c) The line integral of this vector from point P to point Q is independent of the path taken
(d) This vector can represents the magnetic field of some current distribution.

7. A vector $\vec{A} = (5x + 2y)\hat{\imath} + (3y - z)\hat{\jmath} + (2x - az)\hat{k}$ is solenoidal if the constant a has a value:

(a) 4 (b) -4 (c) 8 (d) -8

8. Which of the following vectors is orthogonal to the vector $(a\hat{\imath} + b\hat{\jmath})$, where a and b $(a \neq b)$ are constants and $\hat{\imath}$ and $\hat{\jmath}$ are unit orthogonal vectors?

(a) $-b\hat{\imath} + a\hat{\jmath}$ (b) $-a\hat{\imath} + b\hat{\jmath}$ (c) $-a\hat{\imath} - b\hat{\jmath}$ (d) $-b\hat{\imath} - a\hat{\jmath}$

9. Given any three non-zero vectors $\vec{A}, \vec{B}$ and $\vec{C}$, their triple product. $\vec{A}.(\vec{B} \times \vec{C})$ vanishes if.

(a) They are perpendicular to each other (b) Any two them are perpendicular
(c) Any two of them are parallel (d) They are non-coplanar

10. The necessary and sufficient condition that $\oint_C \vec{A}.d\vec{r} = 0$, for any closed curve C is.
(a) $\vec{\nabla}.\vec{A} = 0$ (b) $\vec{\nabla} \times \vec{A} = 0$ (c) $\vec{\nabla}.\vec{\nabla} \times \vec{A} = 0$ (d) $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = 0$

11. Any arbitrary vector in a three dimensional Cartesian space can be expressed as a linear combination of the following number of linearly independent vectors:
(a) Arbitrary number (b) 1
(c) 2 (d) 3

12. The line integral of $\vec{A}$ vanishes about every closed path. Then $\vec{A}$ must be equal to.
(a) Curl of a vector function (b) Gradient of a scalar function
(c) Gradient of a vector function (d) Zero

13. The vector perpendicular to $3\hat{i} + 4\hat{j} - 5\hat{k}$ is:
(a) $-2\hat{i} + 4\hat{j} - 2\hat{k}$ (b) $2\hat{i} + 4\hat{j} - 2\hat{k}$
(c) $2\hat{i} - 4\hat{j} + 2\hat{k}$ (d) $2\hat{i} - 4\hat{j} - 2\hat{k}$

14. The value of the integral $I = \int_S \vec{r}.d\vec{s}$ where S is the surface enclosing the volume V is:
(a) 3 (b) V (c) 3V (d) 0

15. Identify the CORRECT statements for the following vectors $\vec{a} = 3\hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$.
(a) The vector $\vec{a}$ and $\vec{b}$ are linearly independent.
(b) The vector $\vec{a}$ and $\vec{b}$ are linearly dependent.
(c) The vector $\vec{a}$ and $\vec{b}$ are orthogonal.
(d) The vector $\vec{a}$ and $\vec{b}$ are normalized.

LEVEL-2

16. The value of t for which three vectors.
$[(1-t), 0,0], [1, (1-t), 0] \& [1,1(1-t)]$ are linearly dependent is:
(a) 1 (b) 0 (c) 2 (d) 0

17. Given the four vectors, $u_1 = \begin{pmatrix}1\\2\\1\end{pmatrix}, u_2 = \begin{pmatrix}3\\-5\\1\end{pmatrix}, u_3 = \begin{pmatrix}2\\4\\-8\end{pmatrix}, u_4 = \begin{pmatrix}3\\6\\-12\end{pmatrix}$, the linearly dependent pair is:
(a) u_1, u_2 (b) u_1, u_3 (c) u_1, u_4 (d) u_3, u_4

18. The value of $\oint_S \frac{\vec{r}.d\vec{S}}{r^3}$, where $\vec{r}$ is the position vector and S is a closed surface enclosing the origin is:

(a) 0 (b) π (c) 4 π (d) 8 π

19. Given the four vectors, $u_1 = \begin{pmatrix}1\\2\\1\end{pmatrix}, u_2 = \begin{pmatrix}3\\-5\\1\end{pmatrix}, u_3 = \begin{pmatrix}2\\4\\-8\end{pmatrix}, u_4 = \begin{pmatrix}3\\6\\-12\end{pmatrix}$, the linearly dependent pair is:

(a) u_1, u_2 (b) u_1, u_3 (c) u_1, u_4 (d) u_3, u_4

20. The unit normal to the curve x^2 y^2+xy = 17 at the point (2,0) is:

(a) $\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$ (b) $-\hat{i}$ (c) $-\hat{j}$ (d) $\hat{j}$

21. The two vectors $\vec{P} = \hat{i}, \vec{q} = (\hat{i}+\hat{j})/\sqrt{2}$ are.

(a) Related by a rotation

(b) Related by a reflection through the xy-plane.

(c) Related by an inversion

(d) Not linearly independent

22. The unit vector normal to the surface $3x^2 + 4y = z$ at the point (1,1,7) is:

(a) $(-6\hat{i} + 4\hat{j} + \hat{k})/\sqrt{53}$ (b) $(4\hat{i} + 6\hat{j} - \hat{k})/\sqrt{53}$

(c) $(6\hat{i} + 4\hat{j} - \hat{k})/\sqrt{53}$ (d) $(4\hat{i} + 6\hat{j} + \hat{k})/\sqrt{53}$

23. Consider the set of vectors $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$

(a) The three vectors are othonormal

(b) The three vectors are linearly independent

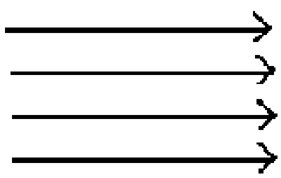
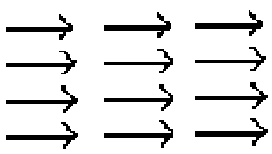
(c) The three vectors cannot form a basis in a three dimension real vectors space

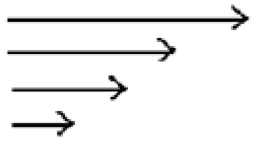
(d) $\frac{1}{\sqrt{2}}(1,1,0)$ Can be written as the linear combination of the following is true?

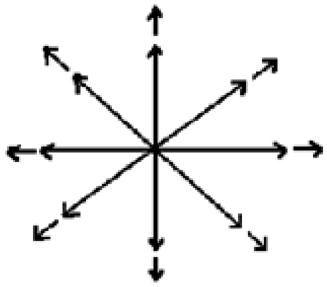
24. If $\vec{A}(t)$ is a vector of constant magnitude, which of the following is true?

(a) $\frac{d\vec{A}}{dt} = 0$ (b) $\frac{d^2\vec{A}}{dt^2} = 0$ (c) $\frac{d\vec{A}}{dt}.\vec{A} = 0$ (d) $\frac{d\vec{A}}{dt} \times \vec{A} = 0$

25. Identify the vector field given below which has a finite curl.

(a)  **(b)** 

(c) 

(d) 

26. If $\vec{A}$ and $\vec{B}$ are two unit vectors and $\theta \neq 0$ is the angle between them, then

(a) $\sin\theta = \frac{1}{2}|\vec{A} + \vec{B}|$
(b) $\sin\theta = \frac{1}{2}|\vec{A} - \vec{B}|$
(c) $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{A} - \vec{B}|$
(d) $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{A} + \vec{B}|$

27. The unit vector normal to the surface $x^2 + 4y = z$ at the point (1,1,0) is:

(a) $\frac{1}{\sqrt{21}}(2\hat{i} + 4\hat{j} - \hat{k})$
(b) $\frac{1}{\sqrt{21}}(-2\hat{i} + 4\hat{j} - \hat{k})$
(c) $\frac{1}{\sqrt{21}}(2\hat{i} - 4\hat{j} - \hat{k})$
(d) $\frac{1}{\sqrt{21}}(2\hat{i} + 4\hat{j} + \hat{k})$

28. Which one of the following vectors is normal to the surface $x^2y + z^3 + xz^2 = 10$?

(a) $x^2 y\hat{i} + z^3\hat{j} + xz^2\hat{k}$
(b) $2xy\hat{i} + 2xz\hat{k}$
(c) $2y\hat{i} + (6z + 2x)\hat{k}$
(d) $(2xy + z^2)\hat{i} + x^2\hat{j} + (3z^2 + 2xz)\hat{k}$

29. Given the vector $\vec{A}(y, -x, 0)$, the line integral $\oint_C \vec{A}.\vec{dl}$, where C is a circle of radius 5 units with its centre at the origin (correct to the first decimal place) is:

(a) 172.8
(b) 157.1
(c) -146.3
(d) 62.8

30. A vector perpendicular to any vector that lies on the plane defined by x+y+z = 5, is

(a) $\vec{i} + \vec{j}$
(b) $\vec{j} + \vec{k}$
(c) $\vec{i} + \vec{j} + \vec{k}$
(d) $2\vec{i} + 3\vec{j} + 5\vec{k}$

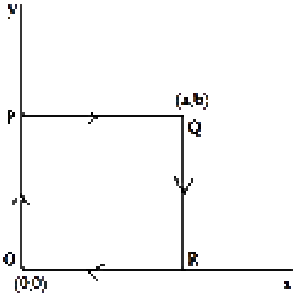
31. If $A = \hat{i}yz + \hat{j}xz + \hat{k}xy$, then the integral $\oint_C A.dl$ (where C is along the perimeter of a rectangular area bounded by x = 0, x = a and y = 0, y = b) is.

(a) $\frac{1}{2}(a^2 + b^2)$
(b) $\pi(ab^3 + a^2b)$
(c) $\pi(a^3 + b^3)$
(d) 0

32. When a force $\vec{F} = (2x + y^2)\hat{\imath} + (3y - 4x)\hat{\jmath}$ is applied on a particle starts to move along a right angled triangles PQR having vertices P (0,0), Q (2,0), R(2,1) respectively, the amount of work done by the force is equal to.

(a) 14/3
(b)-14/3
(c)22/3
(d)-22/3

33. An object is moving along the path O $\rightarrow P \rightarrow Q \rightarrow R \rightarrow O$(shown in the figure) under the force field given as following: F (x,y) = $(x^2 - y^2)\hat{\imath} + 2xy\hat{\jmath}$. The work done by the force field will be.



(a) $-2ab^2$ units (b)$2ab^2$ units (c)$-ab^2$ units (d)ab^2 units

34. If a force field $\vec{A} = (y + 2x)\hat{\imath} + (3x + 2y)\hat{\jmath}$ is applied to a particle, then the work done by the force field in traversing the particle around a circle C in x-y plane, with center at the origin and radius 2units is equal to (C is traverse in the counter-clockwise direction).

(a) 2π (b)4π (c)8π (d)16π

35. If $\vec{F} = (5xy - 6x^2)\hat{\imath} + (2y - 4x)\hat{\jmath}$, then the value of the line integral $\int_C \vec{F}.d\vec{r}$ along the curve C in the x-y plane given by $y = x^3$, from the point (1,1) to (2,8) is equal to.

(a) 35 (b) -35 (c) 47 (d) -47

36. The value of the integral $\iint_S \vec{A}.d\vec{s}$, where $\vec{A} = z\hat{\imath} + x\hat{\jmath} - 3y^2z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between z = 0 and z = 5 (bounded by z = 0 and y = 0 plane), is (in the units of π).

(a) 80 (b) 160 (c) 240 (d) 960

GATE – PREVIOUS YEARS QUESTIONS

1. If S is the closed surface enclosing a volume V and $\hat{n}$ is the unit normal vector to the surface and $\vec{r}$ is the positive vector, then the value of the following integral $\iint_S \vec{r}.\hat{n}dS$ is: **[GATE-2001]**

(a) V (b) 2V (c) 0 (d) 3V

2. Consider the set of vector $\frac{1}{\sqrt{2}}(1,1,0)$, $\frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$. **[GATE-2001]**

(a) The three vectors are orthonormal

(b) The three vectors are linearly independent

(c) The Three vector cannot form a basis in a three-dimensional real vector space.

(d) $\frac{1}{\sqrt{2}}(1,1,0)$ can be written as the linear combination of $\frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$.

3. If $\vec{A} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$, then $\nabla^2\vec{A}$ equals. **[GATE-2001]**
(a) 1 (b) 3 (c) 0 (d) -3

4. A vector $\vec{A} = (5x + 2y)\hat{\imath} + (3y - z)\hat{\jmath} + (2x - az)\hat{k}$ is solenoidal if the constant a has a value: **[GATE-2002]**
(a) 4 (b) -4 (c) 8 (d) -8

5. Which of the following vectors is orthogonal to the vector $(a\hat{\imath} + b\hat{\jmath})$, where a and b (a$\neq b$) are constants, and $\hat{\imath}$ and $\hat{\jmath}$ are unit orthogonal vectors? **[GATE-2002]**
(a) -b$\hat{\imath} + a\hat{\jmath}$ (b) -a$\hat{\imath} + b\hat{\jmath}$ (c) -a$\hat{\imath} - b\hat{\jmath}$ (d) -b$\hat{\imath} - a\hat{\jmath}$

6. The unit vector normal to the surface $3x^2 + 4y = z$ at the point (1,1,7) is: **[GATE-2002]**
(a) $(-6\hat{\imath} + 4\hat{\jmath} + \hat{k})/\sqrt{53}$ (b) $(4\hat{\imath} + 6\hat{\jmath} - \hat{k})/\sqrt{53}$
(c) $(6\hat{\imath} + 4\hat{\jmath} - \hat{k})/\sqrt{53}$ (d) $(4\hat{\imath} + 6\hat{\jmath} + \hat{k})/\sqrt{53}$

Common data for Q.7 & Q.8 -

Consider the vector $\vec{V} = \frac{\vec{r}}{r^3}$. **[GATE-2003]**

7. The surface integral of this vector over the surface of a cube of size and centered at the origin.
(a) 0 (b) 2π (c) 2π a^3 (d) 4π

8. Which one of the following is NOT correct?
(a) Value of the line integral of this vector around any closed curve is zero.
(b) This vector can be written as the gradient of some scalar function.
(c) The line integral of this vector form point P to point Q is independent of the path taken.
(d) This vector can represent the magnetic field of some current distribution.

9. The curl of the vector $\vec{A}$=z$\hat{\imath} + x\hat{\jmath} + y\hat{k}$ is given by: **[GATE-2003]**
(a) $\hat{\imath} + \hat{\jmath} + \hat{k}$ (d) $\hat{\imath} - \hat{\jmath} + \hat{k}$ (c) $\hat{\imath} + \hat{\jmath} - \hat{k}$ (d) $-\hat{\imath} - \hat{\jmath} - \hat{k}$

10. The two vector $\vec{P} = \hat{\imath}, \vec{q} = (\hat{\imath} + \hat{\jmath})/\sqrt{2}$ are: **[GATE-2004]**
(a) Related by a rotation
(b) Related by a reflection through the xy-plane
(c) Related by an inversion (d) Not linearly independent

11. For the function $\phi = x^2y + xy$ the value of $|\vec{\nabla}_\phi|$ at x = y = 1is. **[GATE-2004]**

(a) 5 (b) $\sqrt{5}$ (c) 13 (d) $\sqrt{13}$

12. If a vector field $\vec{F} = x\hat{\imath} + 2y\hat{\jmath} + 3z\hat{k}$, then $\vec{\nabla} \times (\vec{\nabla} \times \vec{F})$ is. **[GATE-2005]**

(a) 0 (b) $\hat{\imath}$ (c) $2\hat{\jmath}$ (d) $3\hat{k}$

13. Given the four vector, $u_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}, u_3 = \begin{pmatrix} 2 \\ 4 \\ -8 \end{pmatrix}, u_4 = \begin{pmatrix} 3 \\ 6 \\ -12 \end{pmatrix}$,the linearly dependent pair is: **[GATE-2005]**

(a) u_1, u_2 (b) u_1, u_3 (c) u_1, u_4 (d) u_3, u_4

14. The unit normal to the curve $x^3y^2 + xy = 17$ at the point (2,0) is: **[GATE-2005]**

(a) $\frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}}$ (b) $-\hat{\imath}$ (c) $-\hat{\jmath}$ (d) $\hat{\jmath}$

15. A vector field is defined everywhere as $\vec{F} = \frac{y^2}{L}\hat{\imath} + z\hat{k}$. The net flux of $\vec{F}$ associated with a cube of side L, with one vertex at the origin and sides along the positive X,Y and Z axes, is: **[GATE-2007]**

(a) L^3 (b) $4L^3$ (c) $8L^3$ (d) $10L^3$

16. If $\vec{r} = x\hat{\imath} + y\hat{\jmath}$, then: **[GATE-2007]**

(a) $\vec{\nabla}.\vec{r} = 0$ and $\vec{\nabla}\,|\vec{r}| = \vec{r}$ (b) $\vec{\nabla}.\vec{r} = 2$ and $\vec{\nabla}\,|\vec{r}| = \vec{r}$

(c) $\vec{\nabla}.\vec{r} = 2$ and $\vec{\nabla}\,|\vec{r}| = \frac{\vec{r}}{r}$ (d) $\vec{\nabla}.\vec{r} = 3$ and $\vec{\nabla}\,|\vec{r}| = \frac{\vec{r}}{r}$

17. Consider a vector $\vec{p} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ in the coordinate system$(\hat{\imath}, \hat{\jmath}, \hat{k})$. The axes are rotated anti-clockwise about the Y axis by an angle of 60^0. the vector $\vec{p}$ in the rotated coordinate system $(\hat{\imath}', \hat{\jmath}', \hat{k}')$ is. **[GATE-2007]**

(a) $(1-\sqrt{3})\hat{\imath}' + 3\hat{\jmath}' + (1+\sqrt{3})\,\hat{k}'$

(b) $(1+\sqrt{3})\hat{\imath}' + 3\hat{\jmath}' + (1-\sqrt{3})\,\hat{k}'$

(c) $(1-\sqrt{3})\hat{\imath}' + (3+\sqrt{3})\hat{\jmath}' + 2\,\hat{k}'$

(d) $(1-\sqrt{3})\hat{\imath}' + (3-\sqrt{3})\hat{\jmath}' + 2\,\hat{k}'$

18. The curl of the vector field is $\vec{F}$ is $2\hat{x}$. Identity the appropriate vector field $\vec{F}$ from the choices given below: **[GATE-2008]**

(a) $\vec{F} = 2z\hat{x} + 3z\hat{y} + 5y\hat{z}$ (b) $\vec{F} = 3z\hat{y} + 5y\hat{z}$

(c) $\vec{F} = 3x\hat{y} + 5y\hat{z}$ (d) $\vec{F} = 2x\hat{y} + 5y\hat{z}$

19. The value of the contour integral $\left|\int_C \vec{r} \times d\vec{\theta}\right|$, for a circle C of radius r with centre at the origin is. **[GATE-2009]**

(a) $2\pi r$ (b) $\frac{r^2}{2}$ (c) πr^2 (d) r

20. An electrostatic field $\vec{E}$ exists in a given region R. Choose the wrong statement: **[GATE-2009]**

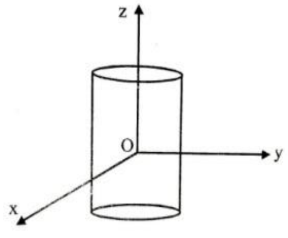
(a) Circulation of $\vec{E}$ is zero.
(b) $\vec{E}$ Can always be expressed as the gradient of a scalar field.
(c) The potential difference between any two arbitrary points in the region R is zero.
(d) The work done in a closed path lying entirely in R is zero.

21. Consider the set of vector in three-dimensional real vector space R^3, S = {(1,1,1),(1-1,1),(1,1-1)}. Which of the following statement is true? **[GATE-2009]**
(a) S is not a linearly independent set
(b) S is a basis for R^3
(c) The vector in S are orthogonal
(d) An orthogonal set of vectors cannot be Generated form S

22. If a force $\vec{F}$ is derivable from a potential function V(r), where r is the distance from the origin of the coordinate system, it follows that. **[GATE-2011]**
(a) $\vec{\nabla} \times \vec{F} = 0$ (b) $\vec{\nabla}.\vec{F} = 0$ (c) $\vec{\nabla}$ V = 0 (d) $\nabla^2 V = 0$

23. The unit vector perpendicular to the surface $x^2 + y^2 - z^2 = 1$ at the point (1,1,1) is . **[GATE-2011]**
(a) $\frac{\hat{\imath}+\hat{\jmath}-\hat{k}}{\sqrt{3}}$ (b) $\frac{2\hat{\imath}+\hat{\jmath}-\hat{k}}{\sqrt{6}}$ (c) $\frac{\hat{\imath}+2\hat{\jmath}-\hat{k}}{\sqrt{6}}$ (d) $\frac{2\hat{\imath}+2\hat{\jmath}-\hat{k}}{3}$

24. Consider a cylinder of height h and radius a, closed at both ends, centered at the origin, Let $\vec{r} = \hat{\imath}x + \hat{\jmath}y + \hat{k}z$ be the position vector and $\hat{n}$ a unit vector normal to the surface. The surface integral $\int_s \vec{r}.\hat{n}ds$ over the closed surface of the cylinder is: **[GATE-2011]**

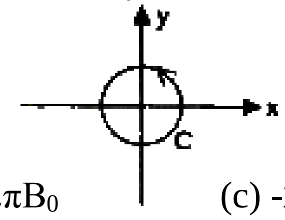


(a) $2\pi a^2(a + h)$ (b) $3\pi a^2 h$ (c) $2\pi a^2 h$ (d) Zero

25. Identify the CORRECT statement for the following vectors $\vec{a} = 3\hat{\imath} + 2\hat{\jmath}$ and $\vec{b} = \hat{\imath}+2\hat{\jmath}$. **[GATE-2012]**
(a) The vector $\vec{a}$ and $\vec{b}$ are linearly independent.

(b) The vector $\vec{a}$ and $\vec{b}$ are linearly dependent.
(c) The vector $\vec{a}$ and $\vec{b}$ are orthogonal
(d) The vector $\vec{a}$ and $\vec{b}$ are normalized.

26. Given $\vec{F} = \vec{r} \times \vec{B}$, where $\vec{B} = B_0\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right)$ is a constant vector and $\vec{r}$ is the position vector. The value of $\oint_c \vec{F}.d\vec{r}$, where C is a circle of unit radius centered at origin is, **[GATE-2012]**



(a) 0 (b) $2\pi B_0$ (c) $-2\pi B_0$ (d) 1

27. If $\vec{A}$ and $\vec{B}$ are constant vectors, then $\vec{\nabla}\left[\vec{A}.(\vec{B} \times \vec{r})\right]$ is. **[GATE-2013]**
(a) $\vec{A}.\vec{B}$ (b) $\vec{A} \times \vec{B}$ (c) $\vec{r}$ (d) zero.

28. The unit vector perpendicular to the surface $x^2 + y^2 + z^2 = 3$ at the point (1,1,1) is. **[GATE-2014]**
(a) $\frac{\hat{x}+\hat{y}-\hat{z}}{\sqrt{3}}$ (b) $\frac{\hat{x}-\hat{y}-\hat{z}}{\sqrt{3}}$ (c) $\frac{\hat{x}-\hat{y}+\hat{z}}{\sqrt{3}}$ (d) $\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$

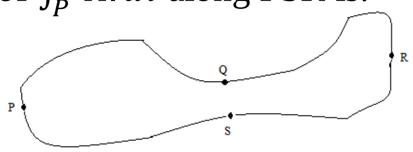
29. Four forces are given below in Cartesian and spherical polar coordinates. **[GATE-2015]**

(a) $\vec{F}_1 = K\, exp\left(\frac{-r^2}{R^2}\right)\hat{r}$ (b) $\vec{F}_2 = K\,(x^3\hat{y} - y^3\hat{z})$

(c) $\vec{F}_3 = K\,(x^3\hat{x} + y^3\hat{y})$ (d) $\vec{F}_4 = K\left(\frac{\hat{\phi}}{r}\right)$

Where K is a constant. Identify the correct option.
(a) (iii) and (iv) are conservative but (i) and (ii) are not
(b) (i) and (ii) are conservative but (iii) and (iv) are not
(c) (ii) and (iii) are conservative but (i) and (iv) are not
(d) (i) and (iii) are conservative but (ii) and (iv) are not

30. Given the magnetic flux through the closed loop PQRSP isϕ. If $\int_P^R \vec{A}.\vec{dl} = \phi_1$ along PQR, the value of $\int_P^R \vec{A}.\vec{dl}$ along PSR is. **[GATE-2015]**



(a) $\phi - \phi_1$ (b) $\phi_1 - \phi$ (c) $-\phi_1$ (d) ϕ_1

31. The direction of $\vec{\nabla}f$ for a scalar field (x,y,z) $= \frac{1}{2}x^2 - xy + \frac{1}{2}z^2$ at the point P (1,1,3) is. **[GATE-2016]**

(a) $\frac{(-\hat{j}-2\hat{k})}{\sqrt{5}}$ (b) $\frac{(-\hat{j}+2\hat{k})}{\sqrt{5}}$ (c) $\frac{(\hat{j}-2\hat{k})}{\sqrt{5}}$ (d) $\frac{(\hat{j}+2\hat{k})}{\sqrt{5}}$

32. In spherical polar coordinate (r,θ,ϕ), the unit vector $\hat{\theta}$ at $\left(10,\frac{\pi}{4},\frac{\pi}{2}\right)$ is. **[GATE-2018]**

(a) $\hat{k}$ (b) $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$ (c) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ (d) $\frac{1}{\sqrt{2}}(\hat{j}-\hat{k})$

33. Given: $\vec{V}_1=\hat{\imath}-\hat{j}$ and $\vec{V}_2=-2\hat{\imath}+3\hat{j}+2\hat{k}$, which one of the following $\vec{V}_3$ makes $(\vec{V}_1,\vec{V}_2,\vec{V}_3)$ a complete set for a three dimensional real linear vector space? **[GATE-2018]**

(a) $\vec{V}_3=\hat{\imath}+\hat{j}+4\hat{k}$ (b) $\vec{V}_3=2\hat{\imath}-\hat{j}+2\hat{k}$
(c) $\vec{V}_3=\hat{\imath}+2\hat{j}+6\hat{k}$ (d) $\vec{V}_3=2\hat{\imath}+\hat{j}+4\hat{k}$

CSIR-NET – PREVIOUS YEARS QUESTIONS

34. Let $\vec{a}$ and $\vec{b}$ be two distinct three-dimensional vector. Then the component of $\vec{b}$ that is perpendicular to $\vec{a}$ given by. **[CSIR JUN-2011]**

(a) $\frac{\vec{a}\times(\vec{b}\times\vec{a})}{a^2}$ (b) $\frac{\vec{b}\times(\vec{a}\times\vec{b})}{b^2}$ (c) $\frac{(\vec{a}.\vec{b})\vec{b}}{a^2}$ (d) $\frac{(\vec{b}.\vec{a})\vec{a}}{a^2}$

35. The equation of the plane that is tangent to the surface xyz = 8 at the point (1,2,4) is. **[CSIR DEC-2011]**

(a) x +2y + 4z = 12 (b) 4x +2y + z = 12
(c) x +4y + 2z = 12 (d) x + y +z = 7

36. A vector perpendicular to any vector that lies on the plane defined by x + y + z = 5, is **[CSIR JUN-2012]**

(a) $\vec{i}+\vec{j}$ (b) $\vec{j}+\vec{k}$ (b) $\vec{i}+\vec{j}+\vec{k}$ (d) $2\vec{i}+3\vec{j}+5\vec{k}$

37. The unit normal vector at the point $\left(\frac{a}{\sqrt{3}},\frac{b}{\sqrt{3}},\frac{c}{\sqrt{3}}\right)$ on the surface of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, is **[CSIR DEC-2012]**

(a) $\frac{bc\hat{\imath}+ca\hat{j}+ab\hat{k}}{\sqrt{a^2b^2+b^2c^2+c^2a^2}}$ (b) $\frac{a\hat{\imath}+b\hat{j}+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$
(c) $\frac{b\hat{\imath}+c\hat{j}+a\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (d) $\frac{\hat{\imath}+\hat{j}+\hat{k}}{\sqrt{3}}$

38. A unit vector $\hat{n}$ on the xy-plane is at an angle of 120^0 with respect to $\hat{\imath}$. The angle between the vectors $\vec{u}=a\hat{\imath}+b\hat{n}$ and $\vec{v}=a\hat{n}+b\hat{\imath}$ will be 60^0 if. **[CSIR JUN-2013]**

(a) b = $\sqrt{3}a/2$ (b) b = 2a /$\sqrt{3}$ (c) b = a/2 (d) b = a

39. If $\vec{A}$ = yz$\hat{\imath}$ + $xz\hat{\jmath} + xy\hat{k}$, then the integral $\oint_C \vec{A}.d\vec{l}$ (where C is along the perimeter of a rectangular area bounded by x = 0, x = a and y = 0, y = b) is.

[CSIR DEC-2013]

(a) $\frac{1}{2}(a^3 + b^3)$ (b) $\pi(ab^2 + a^2b)$

(c) $\pi(a^3 + b^3)$ (d) 0

40. If $\vec{A}$ = yz$\hat{\imath}$ + $zx\hat{\jmath} + xy\hat{k}$ and C is the circle of unit radius in the plane defined by z =1, with the centre on the z-axis, then the value of the integral $\oint_C \vec{A}.d\vec{l}$ is.

[CSIR JUN-2014]

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0

41. Let $\vec{r}$ be the position vector of any point in three dimensional space and r = $|\vec{r}|$.Then

[CSIR DEC-2014]

(a) $\vec{\nabla}.\vec{r} = 0$ and $\vec{\nabla} \times \vec{r} = \frac{\vec{r}}{r}$ (b) $\vec{\nabla}.\vec{r} = 0$ and $\vec{\nabla}^2\vec{r} = 0$

(c)) $\vec{\nabla}.\vec{r} = 3$ and $\vec{\nabla}^2\vec{r} = \frac{\vec{r}}{r^2}$ (d) $\vec{\nabla}.\vec{r} = 3$ and $\vec{\nabla} \times \vec{r} = 0$

42. Consider the three vectors $\vec{v}_1 = 2\hat{\imath} + 3\hat{k}, \vec{v}_2 = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{v}_3 = 5\hat{\imath} + \hat{\jmath} + \alpha\hat{k}$, where $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are the standard unit vectors in a three dimensional Euclidean space. These vectors will be linearly dependent if the value of α is.

[CSIR DEC-2018]

(a) 31/4 (b) 23/4 (c) 27/4 (d) 0

JEST – PREVIOUS YEARS QUESTIONS

43. The vector field xz$\hat{\imath}$ + $y\hat{\jmath}$ in cylindrical polar coordinates is. **[JEST-2013]**

(a) $\rho(z\, cos^2\phi + sin^2\phi)\hat{e}_\rho + \rho\, sin\phi\, cos\phi\, (1 - z)\hat{e}_\phi$

(b) $\rho(z\, cos^2\phi + sin^2\phi)\hat{e}_\rho + \rho\, sin\phi\, cos\phi\, (1 + z)\hat{e}_\phi$

(c) $\rho(z\, sin^2\phi + cos^2\phi)\hat{e}_\rho + \rho\, sin\phi\, cos\phi\, (1 + z)\hat{e}_\phi$

(d) $\rho(z\, sin^2\phi + cos^2\phi)\hat{e}_\rho + \rho\, sin\phi\, cos\phi\, (1 - z)\hat{e}_\phi$

44. What is the equation of the plane which is tangent to the surface xyz = 4 at the point (1,2,2)?

[JEST-2017]

(a) x + 2y +4z =12 (b) 4x+2y + z = 12

(c) x+4y + z = 0 (d) 2x+y + z = 6

45. Let $\vec{r}$ be the position vector of a point on a closed contour C. What is the value of the line integral $\oint \vec{r}.d\vec{r}$?

[JEST-2019]

(a) 0 (b) 1/2 (c) 1 (d) π

46. Which one of the following vectors lie along the line of intersection of the two planes x + 3y –z = 5 and 2x – 2y + 4z = 3?

[JEST-2019]

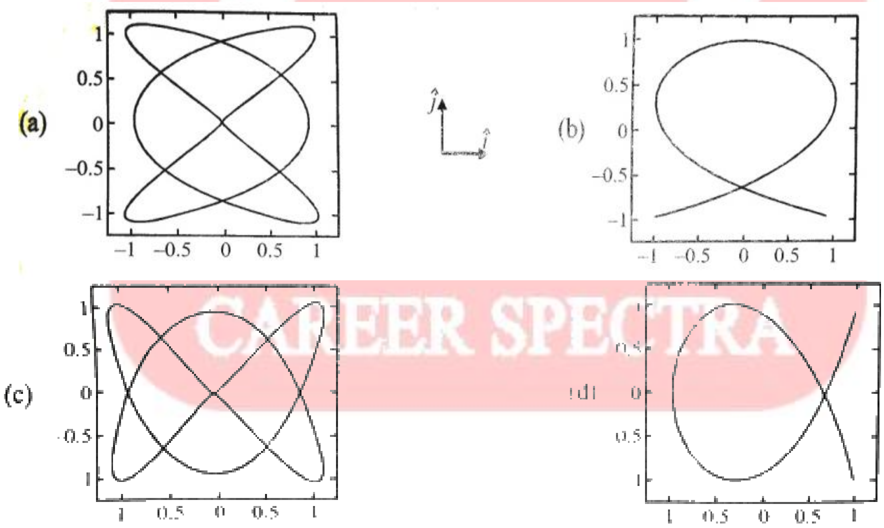
(a) $10\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ (b) $10\hat{\imath} - 6\hat{\jmath} - 8\hat{k}$
(c) $10\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$ (d) $10\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$

47. Suppose $\psi\vec{A}$ is a conservative vector, $\vec{A}$ is non-conservation vector and ψ is non-zero scalar everywhere. Which one of the following is true? **[JEST-2019]**
(a) $(\vec{\nabla} \times \vec{A}).\vec{A} = 0$ (b) $\vec{A} \times \vec{\nabla}\psi = \vec{0}$
(c) $\vec{A}.\vec{\nabla}\psi = 0$ (d) $(\vec{\nabla} \times \vec{A}) \times \vec{A} = \vec{0}$

48. What is the angle (in degrees) between the surface $y^2 + z^2 = 2$ and $y^2 - x^2 = 0$ at the point (1,-1,1)? **[JEST-2019]**

TIFR– PREVIOUS YEARS QUESTIONS

49. A two-dimensional vector $\vec{A}(t)$ is given by $\vec{A}(t) = \hat{\imath}\sin 2t + \hat{\jmath}\cos 3t$.
Which of the following graphs best describes the locus of the tip off the vector, as t is varied from 0 to 2π? **[TIFR-2013]**



50. Consider the surface corresponding to the equation $4x^2 + y^2 + z = 0$. A possible unit tangent to this surface at the point (1,2,-8) is. **[TIFR-2013]**
(a) $\frac{1}{\sqrt{5}}\hat{\imath} - \frac{2}{\sqrt{5}}\hat{\jmath}$ (b) $\frac{1}{5}\hat{\jmath} - \frac{4}{5}\hat{k}$
(c) $\frac{4}{9}\hat{\imath} - \frac{8}{9}\hat{\jmath} + \frac{1}{9}\hat{k}$ (d) $-\frac{1}{\sqrt{5}}\hat{\imath} + \frac{3}{\sqrt{5}}\hat{\jmath} - \frac{4}{\sqrt{5}}\hat{k}$

51. Which of the following vectors is parallel to the surface $x^2y+2xz = 4$ at the point (2,-2,3)? **[TIFR-2015]**
(a) $-6\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ (b) $6\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$
(c) $6\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ (d) $6\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$

52. Consider the surface defined by $ax^2 + by^2 + cz + d = 0$, where a, b, c and d are constants. If $\hat{n}_1$ and $\hat{n}_2$ are unit normal vectors to the surface at the points (x,y,z) = (1,1,0) and (0,0,1) respectively, and $\hat{m}$ is a unit vector normal to both $\hat{n}_1$ and $\hat{n}_2$ then $\hat{m}$ =? **[TIFR-2019]**

(a) $\frac{-a\hat{i}+b\hat{j}}{\sqrt{a^2+b^2}}$ (b) $\frac{a\hat{i}-b\hat{j}+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (c) $\frac{2a\hat{i}+2b\hat{j}-c\hat{k}}{\sqrt{4a^2+4b^2+c^2}}$ (d) $\frac{b\hat{i}-a\hat{j}}{\sqrt{a^2+b^2}}$

ANS-KEY

1	C	2	A	3	A	4	A	5	D
6	D	7	C	8	A	9	C	10	B
11	D	12	B	13	D	14	C	15	A
16	A	17	A	18	C	19	D	20	D
21	A	22	C	23	B	24	C	25	C
26	C	27	A	28	D	29	B	30	C
31	D	32	B	33	A	34	C	35	A
36	A								

PREVIOUS YEAR QUESTIONS ANS-KEY

1	D	13	D	25	A	37	A	49	C
2	B	14	D	26	C	38	C	50	A
3	C	15	A	27	B	39	D	51	C
4	C	16	C	28	D	40	D	52	D
5	A	17	A	29	D	41	D		
6	C	18	B	30	B	42	A		
7	D	19	A	31	B	43	A		
8	D	20	C	32	D	44	D		
9	A	21	B	33	D	45	A		
10	A	22	A	34	A	46	B		
11	D	23	D	35	B	47	A		
12	A	24	B	36	C	48	(60)		